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Where is the Higgs?*

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Abstract

I discuss the theoretical uncertainties in the indirect Higgs mass determination. I present the probability density function for the Higgs mass obtained combining together the information from precision measurements with the results from the direct search experiments carried out at LEP. The probability that the Higgs weighs less than 116 GeV comes out to be around 35% while the 95% upper limit is located around 210-230 GeV.

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1 Introduction

The last months of the year 2000 have seen a great excitement in the physics community because of a possible evidence at LEP of a Higgs boson with a mass $M_H \approx 115$ GeV. Unfortunately the shutdown of LEP will not allow additional information on the Higgs to be collected in the near future. Thus, it seems a good moment to try to review what we (do not) know about the Higgs.

On one side we have the impressive amount of data collected at LEP, SLC, and the Tevatron that allow to probe the quantum structure of the Standard Model (SM), thereby providing indirect information about the Higgs mass. On the other we have the result of the direct searches performed at LEP with the excess of events reported in the combined analysis of the four collaborations. The virtual Higgs effects are usually analyzed through a χ^2 fit to the various precision observables that allows a 95% Confidence Level (C.L.) upper bound to be derived. The outcome of the searches used to be reported as a combined 95% C.L. lower bound, while today is more appropriate described by the likelihood of the experiments. Given these two pieces of information it is often said that one of the greatest achievement of LEP has been to have pin down the Higgs mass between 113 (from the 95% C.L. lower bound of the direct searches) and 170 GeV (from the 95% C.L. upper bound of the global fit to electroweak data). What I would like to discuss in this talk is how confident we are in the fact that $113 \leq M_H \leq 170$ GeV addressing two aspects of this problem: i) how solid is the 170 GeV 95% C.L. upper bound? ii) What is the right way to combine the information from the precision measurements with that from the searches to answer the simple question: “what is the probability that the Higgs mass is between, for example, 113 and 170 GeV?”

2 Theoretical uncertainties in the indirect Higgs mass determination

As well known, the vector boson self-energies are sensitive to virtual Higgs effects and therefore the value of the Higgs mass affects the precision measurements like, for example, the ones made at peak of the Z . However, the Higgs behavior in the relevant electroweak corrections is very mild, just logarithmic. To appreciate how much this logarithmic behavior makes hard to constrain the Higgs let me consider a simple example. Let us take the effective electroweak mixing angle, $\sin^2\theta_{eff}^{lept} \equiv s_{eff}^2$, that is the most important

quantity in the determination of M_H , and write it as

$$s_{eff}^2 \sim (c_1 + \delta c_1) + (c_2 + \delta c_2) \log y; \quad y \equiv (M_H/100 \text{ GeV}). \quad (1)$$

In Eq. (1) I identify the l.h.s. with the experimental result $s_{eff}^2 = (s_{eff}^2)_0 \pm \sigma(s_{eff}^2)$ while in the r.h.s. δc_i represent the theoretical uncertainty in the corresponding coefficients connected to the fact that we have computed c_i in perturbation theory through certain order in the perturbative series and therefore we do not know their exact values because of higher order contributions. From Eq. (1) one obtains

$$y = y_0 \exp \left[-\frac{\Delta_{th}}{c_2} \pm \frac{\sigma}{c_2} \right]; \quad \Delta_{th} = \delta c_1 + \delta c_2 \log y \quad (2)$$

where y_0 is the value corresponding to $\delta c_1 = \delta c_2 = \sigma = 0$. To see the effect of Δ_{th} in extracting M_H I put $\sigma = 0$ and take

$$c_2 \sim \frac{\alpha}{2\pi(c^2 - s^2)} \left(\frac{5}{6} - \frac{3}{4}c^2 \right) \sim 5.5 \times 10^{-4}; \quad \Delta_{th} \sim \pm 1.4 \times 10^{-4} \quad (3)$$

where $s^2 \sim 0.23$, $c^2 = 1 - s^2$. In Eq. (3) I estimate c_2 through the Higgs leading behavior of the correction $\Delta\hat{r}$ relevant for s_{eff}^2 [1] while for Δ_{th} , I take the value estimated in the 1995 CERN report on ‘Precision calculation for the Z resonance’ [2] that was supposed to represent the uncertainty due to next-to-leading two-loop electroweak effects. Inserting the values of Eq. (3) into Eq. (2) yields $y \sim 1.29 y_0$. We see that a theoretical uncertainty coming from two-loop unknown contributions (that are supposed to be not even the dominant part) makes an error in the indirect determination of the Higgs mass of 29 %!

Eq.(2) tells us an important thing, namely that the error on y depends on its central value (y_0). This implies that not always shrinking the uncertainty in $\ln y$ reduces the uncertainty on y . This is true only if the central value does not change, but this is not always the case (as we will see later). This consideration can be put in a more formal way noticing that if the logarithm of a quantity, $A \equiv \ln y$, is normally distributed, then the quantity itself is distributed according to a lognormal (see, e.g., [3] for the properties of this distribution) whose standard deviation, given by

$$\sigma(y) = \left(\exp \left[2E[A] + 2\sigma^2(A) \right] - \exp \left[2E[A] + \sigma^2(A) \right] \right)^{\frac{1}{2}},$$

is a combination of the expected value and standard deviation of its logarithm and therefore compensating effects can happen.

The above example clearly tells us that to extract accurate indirect information on the Higgs one needs both very precise experiments and a very good control of the theory side. This brings in the issue of what error we can associate to our theoretical predictions. They are affected by uncertainties coming from two different sources: one that is called parametric and it is connected to the error in the experimental inputs used in our predictions. The second one is called intrinsic and it is related to the fact that our knowledge of the perturbative series is always limited, usually to the first few terms. Concerning parametric uncertainties, $\alpha(0)$, G_μ and M_Z are very well measured, while the top mass, M_t , and the strong coupling constant, α_s , are not so precisely known. However, the scale of the weak interactions is given by the mass of the intermediate vector bosons, so what actually matters in our predictions is not $\alpha(0)$ but $\alpha(M_Z)$. The latter contains the hadronic contribution to the photon vacuum polarization, $(\Delta\alpha)_h$, that cannot be evaluated in perturbation theory. Fortunately, one can use a dispersion relation to relate it to the experimental data on the cross section for e^+e^- annihilation into hadrons. In the recent years this subject has received a lot of attention with the appearance of several new evaluations that have followed two main streams. On one path there are the most phenomenological (ph.) analyses that rely on the use of all the available experimental data on the hadron production in e^+e^- annihilation and on perturbative QCD (pQCD) for the high energy tail ($E \geq 40$ GeV) of the dispersion integral [4, 5]. On the other the so called “theory driven” (t.d.) analyses that advocate the use of pQCD down to energy scale of the order of tau mass, supplemented by the use of the experimental data in regions, like, for example, the threshold for the charmed mesons, where pQCD is not applicable [6, 7, 8]. I am not going to discuss the differences in the various analyses (see Fred Jegerlehner’s talk [9]) but I would like to point out few facts: i) all results are compatible with each other. The choice of one value instead of another is just a matter of taste (or friendship). ii) The t.d. results have a smaller error with respect to the ph. ones but a lower central value. iii) I can take two perfectly compatible numbers, one from the ph. analyses like $(\Delta\alpha)_h = 0.02804 \pm 0.00065$ [4] and one from the t.d. ones like $(\Delta\alpha)_h = 0.02761 \pm 0.00022$ [8] and get a difference in the 95% C.L. upper bound for the Higgs mass $\mathcal{O}(50 \text{ GeV})$. Just to mention, it is the t.d. value, that has a smaller error, that gives the higher upper bound.

I would like to emphasize that this uncertainty has nothing to do with the blue band in the famous $\Delta\chi^2$ vs. M_H LEPWWG plot. There, the blue band represents, for a chosen value of $(\Delta\alpha)_h$, the intrinsic uncertainty. There is no way to rigorously define the intrinsic uncertainty. The best that

can be done is to try to estimate it by comparing the output of the two codes TOPAZ0 [10] and ZFITTER [11], that now include up to two-loop next-to-leading terms, when they are run enforcing the several built in options for resumming known effects. These options are supposed to mimic the size of unknown higher order terms and the numerical spread in the outputs of the two codes can be taken just as an order of magnitude of the unknown higher order contributions.

3 Higgs mass inference from precision measurements

I am going to discuss now what can we learn about the Higgs from precision measurements. In the spirit of the question raised in the Introduction, I am not going to perform the standard χ^2 analysis, although clearly what I present will be related to it, but following a Bayesian approach I am going to construct $f(m_H | ind.)$, the p.d.f. of the Higgs mass conditioned by this indirect information under the assumption of the validity of the S.M. I would do it employing the three observables, s_{eff}^2 , M_W and Γ_ℓ . These quantities are the most sensitive to the Higgs mass and also very accurately measured. The most convenient way to approach the problem is to make use of the simple parameterization proposed in Ref.[12] and updated in Ref.[13] where s_{eff}^2 , M_W and Γ_ℓ are written as functions of M_H , M_t , α_s and the hadronic contribution to the running of the electromagnetic coupling:

$$s_{eff}^2 = (s_{eff}^2)_o + c_1 A_1 + c_2 A_2 - c_3 A_3 + c_4 A_4, \quad (4)$$

$$M_W = M_W^o - d_1 A_1 - d_5 A_1^2 - d_2 A_2 + d_3 A_3 - d_4 A_4, \quad (5)$$

$$\Gamma_\ell = \Gamma_\ell^o - g_1 A_1 - g_5 A_1^2 - g_2 A_2 + g_3 A_3 - g_4 A_4. \quad (6)$$

In the above equations $A_1 \equiv \ln(M_H/100 \text{ GeV})$, $A_2 \equiv [(\Delta\alpha)_h/0.0280 - 1]$, $A_3 \equiv [(M_t/175 \text{ GeV})^2 - 1]$ and $A_4 \equiv [(\alpha_s(M_Z)/0.118 - 1)]$; $(s_{eff}^2)_o$, M_W^o and Γ_ℓ^o are (to excellent approximation) the theoretical results obtained at the reference point $(\Delta\alpha)_h = 0.0280$, $M_t = 175 \text{ GeV}$, and $\alpha_s(M_Z) = 0.118$ while the values of the coefficients c_i , d_i and g_i are reported in Tables 3-5 of Ref.[13] for three different renormalization scheme. Formulae (4-6) are very accurate for $75 \lesssim M_H \lesssim 350 \text{ GeV}$ with the other parameters in the ranges $170 \lesssim M_t \lesssim 181 \text{ GeV}$, $0.0273 \lesssim (\Delta\alpha)_h \lesssim 0.0287$, $0.113 \lesssim \alpha_s(M_Z) \lesssim 0.123$. In this case they reproduce the exact results of the calculations of Refs.([14, 15, 13]) with maximal errors of $\delta s_{eff}^2 \sim 1 \times 10^{-5}$, $\delta M_W \lesssim 1 \text{ MeV}$ and $\delta \Gamma_\ell \lesssim 3 \text{ KeV}$, which are all very much below the experimental accuracy. Outside the

above range, the deviations increase but remain very small for larger Higgs mass, reaching about 3×10^{-5} , 3 MeV, and 4 KeV at $M_H = 600$ GeV for s_{eff}^2 , M_W , Γ_ℓ , respectively.

Formula (4) can be seen as providing an indirect measurement of $A_1 = [s_{eff}^2 - (s_{eff}^2)_0 - c_2 A_2 + c_3 A_3 - c_4 A_4]/c_1$, while (5) and (6) of the quantities $Y = M_W^0 - M_W - d_2 A_2 + d_3 A_3 - d_4 A_4$ and $Z = \Gamma_\ell^0 - \Gamma_\ell - g_2 A_2 + g_3 A_3 - g_4 A_4$, respectively, all three variables being described by Gaussian p.d.f.'s. The A_1 , Y and Z determination are clearly correlated, therefore one has to build a covariance matrix. This can be easily done because formulae (4–6) are linear in the common terms $\underline{X} \equiv \{M_t, \alpha_s(M_Z), (\Delta\alpha)_h\}$. The likelihood of these indirect measurements $\underline{\Theta} \equiv \{A_1, Y, Z\}$ is then a three dimensional correlated normal with covariance matrix

$$V_{ij} = \sum_l \frac{\partial \Theta_i}{\partial X_l} \cdot \frac{\partial \Theta_j}{\partial X_l} \cdot \sigma^2(X_l) \quad (7)$$

or

$$f(\underline{\theta} | \ln(m_H/100)) \propto e^{-\chi^2/2} \quad (8)$$

where $\chi^2 = \underline{\Delta}^T \mathbf{V}^{-1} \underline{\Delta}$ with $\underline{\Delta}^T = \{a_1 - \ln(m_H/100), y - d_1 \ln(m_H/100) - d_5 \ln^2(m_H/100), z - g_1 \ln(m_H/100) - g_5 \ln^2(m_H/100)\}$. Using Bayes' theorem the likelihood (8) can be turned into a p.d.f. through the choice of a prior. The natural choice is a uniform prior in $\ln(m_H)$, since it is well understood that radiative corrections measure this quantity (for this reason the likelihood (8) has been expressed in terms of $\ln(m_H)$). The uniform prior in $\ln(m_H)$ implies that $f(\ln(m_H/100) | ind.)$ is just the normalized likelihood (8). Thus, using standard probability calculus I can express the result as a p.d.f. of M_H :

$$f(m_H | ind.) = \frac{m_H^{-1} e^{-(\chi^2/2)}}{\int_0^\infty m_H^{-1} e^{-(\chi^2/2)} dm_H}. \quad (9)$$

4 Including the constraint from the direct search

Given $f(m_H | ind.)$ a natural question to ask is how this p.d.f. should be modified in order to take into account the knowledge that the Higgs boson has not been observed (that there is some indication for the Higgs) at LEP¹. To answer this question let me discuss first an ideal case. I consider a search for Higgs production in association with a particle of negligible width in an experimental situation of “infinite” luminosity, perfect efficiency and no

¹A related discussion can be found in Ref.[16].

background whose outcome was no candidate. In this situation we are sure that all mass values below a sharp kinematical limit M_K are excluded. This implies that: a) the p.d.f. for M_H must vanish below M_K ; b) above M_K the relative probabilities cannot change, because there is no sensitivity in this region, and then the experimental results cannot give information over there. For example, if M_K is 110 GeV, then $f(200 \text{ GeV})/f(120 \text{ GeV})$ must remain constant before and after the new piece of information is included. In this ideal case we have then

$$f(m_H | \text{dir. \& ind.}) = \begin{cases} 0 & m_H < M_K \\ \frac{f(m_H | \text{ind.})}{\int_{M_K}^{\infty} f(m_H | \text{ind.}) dm_H} & m_H \geq M_K, \end{cases} \quad (10)$$

where the integral at denominator is just a normalization coefficient.

More formally, this result can be obtained making explicit use of the Bayes' theorem. Applied to our problem, the theorem can be expressed as follows (apart from a normalization constant):

$$f(m_H | \text{dir. \& ind.}) \propto f(\text{dir.} | m_H) \cdot f(m_H | \text{ind.}), \quad (11)$$

where $f(\text{dir.} | m_H)$ is the so called likelihood. In the idealized example we are considering now, $f(\text{dir.} | m_H)$ can be expressed in terms of the probability of observing zero candidates in an experiment sensitive up to a M_K mass for a given value m_H , or

$$f(\text{dir.} | m_H) = f(\text{"zero cand."} | m_H) = \begin{cases} 0 & m_H < M_K \\ 1 & m_H \geq M_K. \end{cases} \quad (12)$$

In fact, we would expect an "infinite" number of events if M_H were below the kinematical limit. Therefore the probability of observing nothing should be zero. Instead, for M_H above M_K , the condition of vanishing production cross section and no background can only yield no candidates.

Consider now a real life situation. In this case the transition between Higgs mass values which are impossible to those which are possible is not so sharp. In fact because of physical reasons (such as threshold effects and background) and experimental reasons (such as luminosity and efficiency) we cannot be really sure about excluding values close to the kinematical limit, nevertheless the ones very far from M_K are ruled out. Furthermore, the kinematical limit is in general not sharp; at LEP, for example, the large total width of the Z^0 plays an important role. Thus, in a real life situation we expect the ideal step function likelihood of Eq. (12) to be replaced by a smooth curve which goes to zero for low masses. Concerning, instead,

the region of no experimental sensitivity, $M_H \gtrsim M_{K_{eff}}$, the likelihood is expected to go to a value independent on the Higgs mass that however is different from that of the ideal case, i.e. 1, because of the presence of the background.

In order to combine the various pieces of information easily it is convenient to replace the likelihood by a function that goes to 1 where the experimental sensitivity is lost [17, 18]. Because constant factors do not play any role in the Bayes' theorem this can be achieved by dividing the likelihood by its value calculated for very large Higgs mass values where no signal is expected, i.e. the case of pure background. This likelihood ratio, \mathcal{R} , can be seen as the counterpart, in the case of a real experiment, of the step function of Eq. (12). Therefore, the Higgs mass p.d.f. that takes into account both direct search and precision measurement results can be written as

$$f(m_H | dir. \& ind.) = \frac{\mathcal{R}(m_H) f(m_H | ind.)}{\int_0^\infty \mathcal{R}(m_H) f(m_H | ind.) dm_H} . \quad (13)$$

In Eq. (13) \mathcal{R} , namely the information from the direct searches, acts as a shape distortion function of $f(m_H | ind.)$. As long as $\mathcal{R}(m_H)$ is 1, the shape (and therefore the relative probabilities in that region) remains unchanged, while $\mathcal{R}(m_H) \rightarrow 0$ indicates regions where the p.d.f. should vanish.

One should notice that $\mathcal{R}(m_H)$ can also assume values larger than 1 for Higgs mass values below the kinematical limit. This situation corresponds to a number of observed candidate events larger than the expected background. In this case the role played by $\mathcal{R}(m_H)$ is to stretch $f(m_H | ind.)$ below the effective kinematical limit and this might even prompt a claim for a discovery if \mathcal{R} becomes sufficiently large for the probability of M_H in that region to get very close to 1.

5 Results

The experimental inputs I use to construct $f(m_H | ind.)$ are [19]: $s_{eff}^2 = 0.23146 \pm 0.00017$, $M_W = 80.419 \pm 0.038$ GeV, $\Gamma_\ell = 83.99 \pm 0.10$ MeV, $M_t = 174.3 \pm 5.9$ GeV, $\alpha_s(M_Z) = 0.119 \pm 0.003$. Concerning $(\Delta\alpha)_h$, as I said we have many possible choices. I am going to present my results using two different values of $(\Delta\alpha)_h$, one as representative of the ph. analyses and the other for the t.d. evaluations. For the ph. case I take the standard value $(\Delta\alpha)_h^{EJ} = 0.02804 \pm 0.00065$ [4] while for the t.d. analyses I choose the one that has the smallest uncertainty, i.e. $(\Delta\alpha)_h^{DH} = 0.02770 \pm 0.00016$ [7]. The intrinsic uncertainty is taken into account by averaging different inferences

	$(\Delta\alpha)_h = 0.02804(65)$		$(\Delta\alpha)_h = 0.02770(16)$	
	$(ind.)$	$\left(\begin{array}{c} ind. \\ + \\ dir. \end{array} \right)$	$(ind.)$	$\left(\begin{array}{c} ind. \\ + \\ dir. \end{array} \right)$
$E[M_H]/\text{GeV}$	90	140	105	135
$\sigma(M_H)/\text{GeV}$	55	45	50	35
$P(M_H \leq 110 \text{ GeV})$	73 %	~ 0	61 %	~ 0
$P(M_H \leq 116 \text{ GeV})$	76 %	37 %	66 %	38 %
$P(M_H \leq 130 \text{ GeV})$	82 %	66 %	75 %	68 %
$P(M_H \leq 200 \text{ GeV})$	95 %	91 %	95 %	94 %
$M_H^{95}/\text{GeV}; P(M_H \leq M_H^{95}) \approx 0.95$	195	230	200	210
$M_H^{99}/\text{GeV}; P(M_H \leq M_H^{99}) \approx 0.99$	290	330	275	280

Table 1: Summary of the direct plus indirect information.

(see Ref.[17]).

The values of the \mathcal{R} function that enters in Eq. (13) has been provided by the LEP Higgs Working Group [20]: they take into account all Higgs searches by the four LEP collaborations.

Table 1 summarizes the result of my analysis. The shape of the p.d.f. with and without the inclusion of the direct search information is presented in Fig. 1. From this figure one can notice that, in the case of $f(m_H | ind.)$, the use of a higher central value for $(\Delta\alpha)_h$ (i.e. $(\Delta\alpha)_h^{EJ}$) tends to concentrate more the probability towards smaller values of M_H . As a consequence the analysis based on $(\Delta\alpha)_h^{EJ}$ gives results for the expected value, $E[M_H]$, the standard deviation, $\sigma(M_H)$, of the M_H p.d.f. and 95 % probability upper limit, M_H^{95} , very close to those obtained using $(\Delta\alpha)_h^{DH}$ regardless the fact that the uncertainty on $(\Delta\alpha)_h^{EJ}$ is approximately 4 times larger than that of $(\Delta\alpha)_h^{DH}$. As expected, the inclusion of the direct search information drifts the p.d.f. towards higher values of M_H by cutting regions below 110 GeV. However, the most striking thing in the figure is the spike around

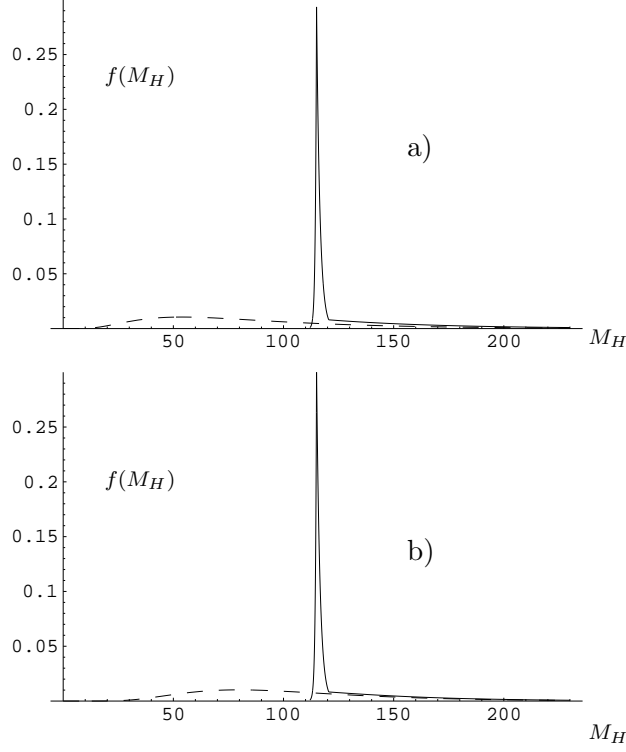


Figure 1: Probability distribution functions using only indirect information (dashed line) and employing also the experimental results from direct searches (solid one): a) $(\Delta\alpha)_h = 0.02804(65)$; b) $(\Delta\alpha)_h = 0.02770(16)$.

$M_H \approx 115$ GeV. Indeed, due to the excess of events in the combined data set of the LEP experiments, the \mathcal{R} function around 115 GeV takes values ~ 30 strongly enhancing the probability of that region.

6 Conclusions

The analysis that I have presented clearly shows that a heavy Higgs scenario is highly disfavored, given a $\mathcal{O}(90\%)$ probability that the Higgs weighs less than 200 GeV. However, it should be said that the distribution has a quite long tail. Indeed we have that above 300 GeV there is still a residual $\mathcal{O}(1\%)$ probability. The excess in the data recorded by the LEP collaborations is reflected in the analysis by the spike in the distribution around $M_H \approx$

115 GeV. Maybe it can be of some interest to know that, according to my analysis, the probability that the Higgs mass is below 116 GeV is 37%, if we use $(\Delta\alpha)_h^{EJ}$, and 38% in the case of $(\Delta\alpha)_h^{DH}$.

Let me say clearly that all the results I have presented are derived under the assumptions of the validity of the SM and rely on the experimental inputs I have used. In particular I have used for s_{eff}^2 the combined LEP+SLD value, although it is well known that the two most precise determinations of it are not in very good agreement. Just to see how much my conclusions rely on the combined s_{eff}^2 value I have redone the analysis discarding the SLD result, i.e. using the LEP value for the effective sine, $s_{eff}^2 = 0.23184 \pm 0.00023$. The results of this extreme exercise are not very different: in the worst case, i.e. employing $(\Delta\alpha)_h^{DH}$, $P(M_H \leq 200 \text{ GeV}) = 75\%$ while $M_H^{95} = 320 \text{ GeV}$. Finally there is still a 23% probability that the Higgs mass is below 116 GeV.

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